University of Bahrain
College of Information Technology
Computer Engineering Department
Semester 1, 2015/2016
Instructor: Dr. Salman Khan

ITCE 380
Name: ………………………
ID: ………………………….
09Novervmber 2015
Duration: 1 hour 15 minutes

Exam # 1 SOLUTION

Question number	Max. Marks	Marks obtained
Q1	10	
Q2	10	
Q3	10	
Q4	10	
Q5	10	
Total	40*	

Instructions:

- * Solve a total of **FOUR** questions. **Question 5 is compulsory**
- DO NOT solve all questions. If you solve all questions, then only the first three questions and Q5 will be graded.
- All calculations should be done to three decimal points, rounded-off
- Show all your work to claim full credit.

Q1 : Solve for the root of $f(x) = x^3 - 3x + 0.5$ with a = 1 and b = 2 using Bisection method. Solve to five iterations.

SOLUTION:

a	c (midpoint)	b	f(a)	f(c)	f(b)
1	1.5	2	-1.500	-0.625	2.500
1.5	1.75	2	-0.625	0.609	2.500
1.5	1.625	1.75	-0.625	-0.084	0.609
1.625	1.688	1.750	-0.084	0.243	0.609
1.625	1.656	1.688	-0.084	0.075	0.243

The root after five iterations is 1.656

Q2 : Use the Newton-Raphson method to estimate the root of $f(x) = e^{-x} - x$ with initial guess of $x_0 = 0$ Approximate until the percent error < 0.100.

Solution. The first derivative of the function can be evaluated as

$$f'(x) = -e^{-x} - 1$$

which can be substituted along with the original function into Eq. (6.6) to give

$$x_{i+1} = x_i - \frac{e^{-x_i} - x_i}{-e^{-x_i} - 1}$$

Starting with an initial guess of $x_0 = 0$, this iterative equation can be applied to compute

i	x_i	$\lvert\varepsilon_t\rvert$, %
0	0	100
1	0.500000000	11.8
2	0.566311003	0.147
3	0.567143165	0.0000220

Q3 : Using the Secant method, find a root of the function $f(x) = 3x + \sin(x) - e^x$. Use $x_0 = 0$ and $x_1 = 1.0$ as initial guesses. Approximate until the absolute error < 0.005.

SOLUTION:

i	0	1	2	3	4	5
x_i	0	1	0.471	0.308	0.363	0.36

Q4: Use the Gauss-Seidel method to solve the following system. Use initial guess of $x_1 = 0$, $x_2 = 0$, and $x_3 = 0$ and conduct three iterations.

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3 \end{aligned}$$

Solution To begin, write the system in the form

$$x_1 = -\tfrac{1}{5} + \tfrac{2}{5}x_2 - \tfrac{3}{5}x_3$$
$$x_2 = \tfrac{2}{9} + \tfrac{3}{9}x_1 - \tfrac{1}{9}x_3$$
$$x_3 = -\tfrac{3}{7} + \tfrac{2}{7}x_1 - \tfrac{1}{7}x_2.$$

using $(x_1, x_2, x_3) = (0, 0, 0)$ as the initial approximation, you obtain the following new value for x_1.

$$x_1 = -\tfrac{1}{5} + \tfrac{2}{5}(0) - \tfrac{3}{5}(0) = -0.200$$

Now that you have a new value for x_1, however, use it to compute a new value for x_2. That is,

$$x_2 = \tfrac{2}{9} + \tfrac{3}{9}(-0.200) - \tfrac{1}{9}(0) \approx 0.156.$$

Similarly, use $x_1 = -0.200$ and $x_2 = 0.156$ to compute a new value for x_3. That is,

$$x_3 = -\tfrac{3}{7} + \tfrac{2}{7}(-0.200) - \tfrac{1}{7}(0.156) \approx -0.508.$$

So the first approximation is $x_1 = -0.200$, $x_2 = 0.156$, and $x_3 = -0.508$. Continued iterations produce the sequence of approximations shown in Table

TABLE

n	1	2	3
x_1	−0.200	0.167	0.191
x_2	0.156	0.334	0.333
x_3	−0.508	−0.429	−0.422

Q5: Use the matrix inversion method to solve the following system of simultaneous equations.

$$\begin{aligned} -x + 3y + z &= 1 \\ 2x + 5y \quad &= 3 \\ 3x + y - 2z &= -2 \end{aligned}$$

<u>Solution</u>

Step 1: Rewrite the system using matrix multiplication:

$$\begin{pmatrix} -1 & 3 & 1 \\ 2 & 5 & 0 \\ 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

and writing the coefficient matrix as A, we have

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}.$$

Step 2: Find the inverse of the coefficient matrix A. In this case the inverse is

$$A^{-1} = \begin{pmatrix} -\frac{10}{9} & \frac{7}{9} & -\frac{5}{9} \\ \frac{4}{9} & -\frac{1}{9} & \frac{2}{9} \\ -\frac{13}{9} & \frac{10}{9} & -\frac{11}{9} \end{pmatrix}$$

Step 3: Multiply both sides of the equation (that you wrote in step #1) by the matrix A^{-1}.

On the left you'll get

$$A^{-1}A\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{which is} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

On the right, you get

$$A^{-1}\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -\frac{10}{9} & \frac{7}{9} & -\frac{5}{9} \\ \frac{4}{9} & -\frac{1}{9} & \frac{2}{9} \\ -\frac{13}{9} & \frac{10}{9} & -\frac{11}{9} \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

Which will result in values x = 7/3, y = -1/3, and z = 13/3